# Justification of Gerber's Gravitational Potential

by Walter Orlov

Paul Gerber recognized two factors that due to finite propagation speed of interaction can influence the Newtonian law of gravitation.

1. Neumann's factor: the potential of the mutual attraction of two masses requires some time to get from one mass to another mass. This correction factor is determined by

$$\frac{1}{1-\frac{v_r}{c}}.$$

There are usually no trouble to understand this effect.

2. Gerber's factor: duration of impact of gravitational potential. According to Gerber, the gravitational interaction has the constant speed (of light) only relative to the mass, from which it emanates. This leads to greater "exposure time" in the removal of the masses of each other and vice versa.

How much time needs a change of field from the mass $m_1$ to run by the mass $m_2$ with diameter $d$ (figure 1)?

At rest:

$$\Delta t_0 = \frac{d}{c}.$$

When running togetherness:

$$\Delta t = \frac{d}{c+v_r}.$$

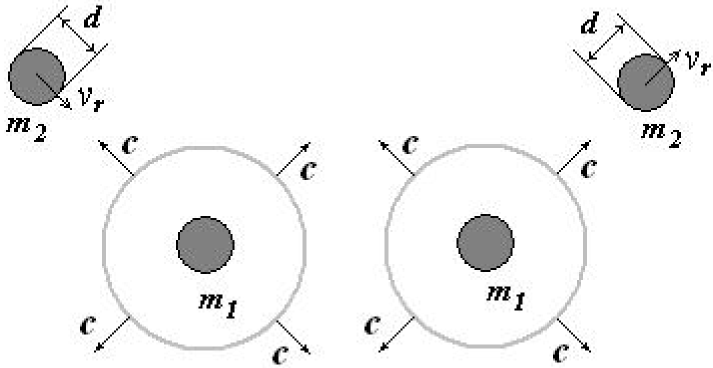


Figure 1. Movement of the masses from
the perspective of the mass $m_1$ .

When running away:

$$\Delta t = \frac{d}{c - v_r} .$$

Obviously the same holds for field change from the mass $m_2$ . It follows:

$$\frac{\Delta t}{\Delta t_0} = \frac{1}{1 - \frac{v_r}{c}} .$$

Taken together, two factors make the potential:

$$V = \frac{\mu}{r\left(1 - \frac{v_r}{c}\right)^2} .$$

This formula allowed Gerber in year 1898 to compute the perihelion precession of the planets.

Gerber's gravitational potential can be represented through binomial series:

$$V=\frac{\mu}{r\left(1-\frac{v_r}{c}\right)^2}=\frac{\mu}{r}\left[1+2\frac{v_r}{c}+3\left(\frac{v_r}{c}\right)^2+...\right].$$

He used general Lagrangian equation of motion:

$$g=\frac{\partial V}{\partial r}-\frac{d}{dt}\frac{\partial V}{\partial v_r},$$

$$g=-\frac{\mu}{r^2}\left[1+2\frac{v_r}{c}+3\frac{v_r^2}{c^2}\right]+\frac{\mu}{r^2}\left[2\frac{v_r}{c}+6\frac{v_r^2}{c^2}-6\frac{r\dot{v}_r}{c^2}\right],$$

$$g=-\frac{\mu}{r^2}\left[1-3\frac{v_r^2}{c^2}+6\frac{r\dot{v}_r}{c^2}\right].$$

Gerber made some calculations and came to

$$r=\frac{\frac{L^2}{\mu m^2}}{1+\left(\int F\sin\vartheta d\vartheta+N\right)\cos\vartheta-\left(\int F\cos\vartheta d\vartheta+M\right)\sin\vartheta}.$$

Here is

$$F=3\frac{v_r^2}{c^2}-6\frac{r\dot{v}_r}{c^2}$$

and $M, N$ are the integration constants. Figure 2 shows the names of variables, which Paul Gerber used. The comparison with the general form of the orbit equation

$$r=\frac{p}{1+\varepsilon\cos\alpha}$$

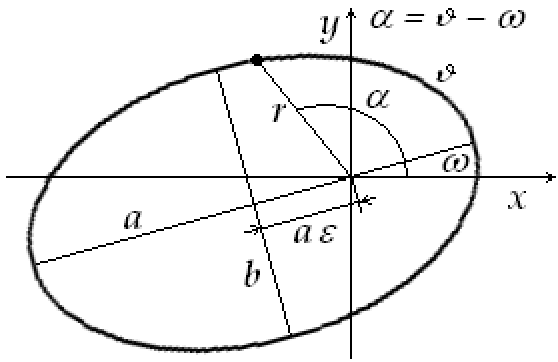


Figure 2. Elliptical planetary orbit with Gerber's namesof variables.

provides

$$F=-\frac{\varepsilon}{\cos\alpha}\frac{d\,\omega}{dt}\frac{dt}{d\,\vartheta}.$$

Now we need to identify from the left side

$$F=\frac{3}{c^2}\left(\frac{dr}{dt}\right)^2-\frac{6r}{c^2}\frac{d^2r}{dt^2}.$$

Gerber makes:

$$F=\frac{3}{c^2}\frac{a\mu}{b^2}\varepsilon^2\sin^2\alpha+\frac{6r}{c^2}\varepsilon\frac{\sqrt{a\mu}}{b\cos\alpha}\frac{d\omega}{dt}-\frac{6}{c^2}\varepsilon\frac{\mu}{r}\cos\alpha$$

It follows:

$$\frac{d\,\omega}{dt}=-\frac{3\sqrt{a}\,\mu^{3/2}}{r^2c^2b}\varepsilon\sin^2\alpha\cos\alpha-\frac{6\mu}{rc^2}\frac{d\,\omega}{dt}+\frac{6\mu^{3/2}b}{r^3c^2\sqrt{a}}\cos^2\alpha.$$

With

$$b=a\sqrt{1-\varepsilon^2}\quad,\quad r=\frac{b^2/a}{1+\varepsilon\cos\alpha}=\frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\alpha}$$

we reduce the number of independent variables

$$\frac{d\,\omega}{dt}=-\frac{3}{c^2}\mu^{3/2}\frac{(1+\varepsilon\cos\alpha)^2}{a^{5/2}\left(1-\varepsilon^2\right)^{5/2}}\varepsilon\sin^2\alpha\cos\alpha$$

$$-\frac{6\mu}{c^2}\frac{(1+\varepsilon\cos\alpha)}{a\left(1-\varepsilon^2\right)}\frac{d\,\omega}{dt}$$

$$+\frac{6\mu^{3/2}}{c^2}\frac{(1+\varepsilon\cos\alpha)^3}{a^{5/2}\left(1-\varepsilon^2\right)^{5/2}}\cos^2\alpha\,.$$

Furthermore

$$dt=\frac{m\,r^2}{L}d\,\vartheta=\frac{m\,r^2}{L}d\,(\alpha+\omega)\,,$$

$$L=m\sqrt{\mu\,a}\sqrt{1-\varepsilon^2}\,,$$

$$dt=\frac{a^{3/2}\left(1-\varepsilon^2\right)^{3/2}}{\sqrt{\mu}(1+\varepsilon\cos\alpha)^2}(d\,\alpha+d\,\omega)\,.$$

With

$$\gamma=\frac{3\mu}{a\left(1+\varepsilon^2\right)}$$

the equation for $d\,\omega$ is

$$d\,\omega=\frac{-\varepsilon\cos\alpha+2\cos^2\alpha+3\,\varepsilon\cos^3\alpha}{\frac{c^2}{\gamma}+2+3\,\varepsilon\cos\alpha-2\cos^2\alpha-3\,\varepsilon\cos^3\alpha}d\,\alpha\,.$$

The integration leads to the perihelion shift per round:

$$\psi = \frac{6\pi\mu}{c^2 a\left(1+\varepsilon^2\right)+6\mu} - \frac{27\mu^2\left(\varepsilon^2-8\right)}{8\left[c^2 a\left(1+\varepsilon^2\right)+6\mu\right]^2}.$$

This formula can be reduced to the first member:

$$\psi \approx \frac{6\pi\mu}{c^2 a\left(1+\varepsilon^2\right)}.$$

18 years after Gerber's publication appeared the same formula in the general theory of relativity.

## Literature


Paul Gerber, Die räumliche und zeitliche Ausbreitung der Gravitation (The Space and Time Propagation of Gravitation). Zeitschrift für Mathematik und Physik. 43, 1898, S. 93–104 http://de.wikisource.org/wiki/Die_r%C3%A4umliche_und_zeitliche_Ausbreitung_der_Gravitation

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Full text in Germany: http://www.walter-orlov.wg.am/buch/